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LETTER TO THE EDITOR

Quantum invariants of motion in a generic many-body system

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Abstract. A dynamical Lie algebraic method for the construction of quantum invariants of motion in non-integrable many-body systems of infinite size is proposed and applied to a simple but generic toy model, namely an infinite kicked t–V chain of interacting spinless fermions. The transition from an *integrable* via quasi-integrable (*intermediate*) to a *quantum ergodic (quantum mixing)* regime in parameter space is investigated. A *dynamical phase transition* between an *ergodic* and *intermediate* (neither ergodic nor completely integrable) regime in thermodynamic limit is proposed. The existence or non-existence of local conservation laws corresponds to the intermediate or ergodic regime, respectively. The computation of time-correlation functions of typical observables by means of local conservation laws is found to be fully consistent with direct calculations on finite systems.

We investigate the existence of local quantum invariants of motion (LQI) (i.e. conservation laws) in a generic quantum many-body system of locally interacting particles (with short-range two-body interactions). For sufficiently strong nonlinear interactions between particles one expects the dynamical properties of quantum mixing and quantum ergodicity to take place in the thermodynamic limit (TL) (size $L \to \infty$ and density of particles ρ fixed) [1], which is in general incompatible with the existence of LQI [2]. Infinite quantum many-body systems will be called *quantum mixing* [1] if for an arbitrary pair of quantum observables in Heisenberg representation, $A(\tau)$ and $B(\tau)$, time correlations $\langle A(0)B(\tau)\rangle - \langle A\rangle\langle B\rangle$ decay to zero as $\tau \to \infty$. Quantum mixing implies *quantum ergodicity*, which in the case of explicitly time-dependent, e.g. kicked systems, such as the one considered in this letter, states that for an arbitrary observable A, the time average $\bar{A}$ equals a unit operator times the Fock space ('micro-canonical') average, $\bar{A} = \langle A\rangle 1$, $\langle A\rangle = \lim_{L\to\infty} \mathrm{tr}\, A/\mathrm{tr}\, 1$. The properties of quantum ergodicity and mixing are necessary for the derivation of the laws of normal transport within the linear response theory. In another extreme case of a completely integrable quantum many-body systems, an infinite number of LQI exist, and such systems are manifestly non-ergodic and hence non-mixing. Anomalous (ideal, ballistic) transport properties of completely integrable quantum many-body systems have recently been discussed [3]. However, the delicate and important issue is the transition from integrable to quantum ergodic (and quantum mixing) dynamics in TL when a certain non-integrability parameter, say λ, is continuously varied (analogous to order-to-chaos transitions of classical few-body systems). Let H_λ denote a *generic* family of locally (*non-randomly*) interacting infinite ($L = \infty$) quantum many-body systems, such that H_λ is completely integrable for $\lambda = 0$, and (almost) ergodic and mixing for sufficiently large λ. Based

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on recent results [4] on time evolution in a particular family of finite many-body systems of increasing size L, we propose a conjecture on the existence of *intermediate*, neither integrable nor ergodic, dynamical regime in TL.

Conjecture. $\exists\lambda_c > 0$ such that H_λ is non-ergodic and non-mixing for $|\lambda| < \lambda_c$.

It is the purpose of this letter to reconfirm this conjecture by an independent approach and to show that such an intermediate regime may be characterized by *quantum quasi-integrability*: the existence of at least one (or more) LQI, which is a sufficient condition, using a relation proposed by Mazur [5] and Suzuki [6], to prevent time correlations of generic observables to decay to zero. We expect that the validity of our conjecture can improve the general understanding of statistical dynamics, and in particular the transport properties of generic quantum many-body systems.

In [4] a novel family of simple but generic many-body systems of locally interacting particles has been introduced smoothly interpolating between integrable and ergodic regime, namely a *kicked t–V model* (KtV) of spinless fermions with periodically switched nearest neighbour interaction on an infinite chain with a time-dependent Hamiltonian

$$H(\tau) = \sum_{j=-\infty}^{\infty} [-\tfrac{1}{2}t(c_j^\dagger c_{j+1} + \text{h.c.}) + \delta_p(\tau)Vn_jn_{j+1}]. \tag{1}$$

$c_j^\dagger, c_j, n_j = c_j^\dagger c_j$ are fermionic creation, annihilation and number operators, respectively, and $\delta_p(\tau) = \sum_{m=-\infty}^{\infty}\delta(\tau - m - \frac{1}{2})$ is a periodic δ-function of period 1 with kicks, for convenience, occurring at half-integer values of time. The time-dependent Hamiltonian (1) can be written as $H(\tau) = tH_1 + \delta_p(\tau)VH_0$ where the dimensionless kinetic energy H_1 and the kick potential H_0 may be rewritten in terms of independent spin-$\frac{1}{2}$ variables $(\sigma_j^\pm, \sigma_j^z)$, $\sigma_j^\pm := (\sigma_j^x \pm \mathrm{i}\sigma_j^y)/\sqrt{2}$, on sites j, via the Wigner–Jordan transformation,

$$H_1 = \tfrac{1}{4}\sum_j(\sigma_j^+\sigma_{j+1}^- + \sigma_j^-\sigma_{j+1}^+) \qquad H_0 = \tfrac{1}{4}\sum_j \sigma_j^z\sigma_{j+1}^z$$

and hence the KtV Hamiltonian (1) may be rewritten as a kicked Heisenberg XXZ chain. Symmetric time evolution of a KtV system for one period is given by an explicit unitary quantum many-body map ($\hbar = 1$), $U = \hat{T}\exp(-\mathrm{i}\int_0^1 \mathrm{d}\tau\, H(\tau)) = \exp(-\mathrm{i}tH_1/2)\exp(-\mathrm{i}VH_0)\exp(-\mathrm{i}tH_1/2)$. Note that V is a cyclic parameter of period 2π, and the dynamics is essentially invariant w.r.t. transformations $t \to -t$ and $V \to -V$, so we consider only the half-strip of parameters $(t, V) \in [0, \infty) \times [0, \pi)$. The KtV model is completely integrable for $t = 0$ (Ising model), or $V = 0 \pmod{2\pi}$ (free fermion model), or $t, V \to 0$ and t/V finite (XXZ model).

Let us consider the Heisenberg representation of quantum dynamics and write a map over the algebra $\mathfrak{U}$ of quantum observables $A(\tau)$ for time evolution over one period,

$$\hat{U}_{\text{ad}} : A(n) \to A(n+1) = U^\dagger A(n)U$$

explicitly as

$$\hat{U}_{\text{ad}} = \exp\left(\frac{\mathrm{i}t}{2}\text{ad}H_1\right)\exp(\mathrm{i}V\text{ad}H_0)\exp\left(\frac{\mathrm{i}t}{2}\text{ad}H_1\right)$$

where ad is the usual adjoint map on the Lie algebra $\mathfrak{U}$, $\text{ad}A : B \to [A, B] = AB - BA$. Infinite-dimensional Lie algebra $\mathfrak{U}$ of local quantum observables constitutes a *Hilbert space* equipped with the *invariant bilinear form*

$$(A|B) = \lim_{L\to\infty}\frac{1}{L2^L}\,\text{tr}_L\, A^\dagger B \tag{2}$$

which is a scalar product invariant under the adjoint map $\mathrm{ad}A : B \to [A, B]$, $((\mathrm{ad}A^\dagger)B|C) = (B|(\mathrm{ad}A)C)$. The trace tr_L refers to a finite chain of finite (though increasing) length L. We should note that the *locality* of an observable A (in a weak sense) is defined here to be equivalent to its normalizability $(A|A) < \infty$ in the metric (2). For example, spatially homogeneous observables of the special form $\sum_j \sum_l a_{j-l} c_j^\dagger c_l$ are local if and only if the sequence a_l is in ℓ_2, i.e. $|a_l|^2 < C/(|l|+1)^\beta$ for some $C > 0$ and $\beta > 1$. Our aim is to check the existence of LQI which are the normalizable fixed points A of the Heisenberg map $\hat{U}_{\mathrm{ad}}$

$$\hat{U}_{\mathrm{ad}} A = A. \tag{3}$$

However, we do not suggest to solve equation (3) in the full algebra $\mathfrak{U}$ which is a highly prohibitive task. Instead, we construct the simplest invariant infinitely dimensional subalgebra of $\mathfrak{U}$, namely the minimal invariant Lie algebra (MILA) $\mathfrak{S}$, which is (by construction) invariant to motions generated by the kinetic or the potential part of the Hamiltonian and hence it is also invariant to $\hat{U}_{\mathrm{ad}}$. Having the two generators, H_0 and H_1, spanning two-dimensional subspace $\mathfrak{s} = \{\alpha H_0 + \beta H_1\}$, we construct the basis of MILA $\mathfrak{S} = \bigcup_{n=1}^{\infty} (\mathrm{ad}\mathfrak{s})^n \mathfrak{s}$ ordered by the order of locality as follows. We assign an observable $\tilde{H}_{p,b}$ to an *ordered pair* of integers (p, b), *order* p, and *code* b, $0 \leqslant b < 2^p$ with p binary digits b_n, $b = \sum_{n=0}^{p-1} b_n 2^n$, namely

$$\tilde{H}_{p,b} = (\mathrm{ad}H_{b_{p-1}})(\mathrm{ad}H_{b_{p-2}}) \ldots (\mathrm{ad}H_{b_1}) H_{b_0}.$$

Since not all observables $\tilde{H}_{p,b}$ up to a given maximal order q, $p \leqslant q$, are linearly independent we perform Gram–Schmit orthogonalization w.r.t. the scalar product (2)

$$G_{q,c} = \begin{cases} \tilde{G}_{q,c}/\sqrt{(\tilde{G}_{q,c}|\tilde{G}_{q,c})} & \tilde{G}_{q,c} \neq 0 \\ 0 & \tilde{G}_{q,c} = 0 \end{cases}$$
$$\tilde{G}_{q,c} = \tilde{H}_{q,c} - \sum_{(p,b)}^{(p,b)<(q,c)} G_{p,b}(G_{p,b}|\tilde{H}_{q,c}). \tag{4}$$

The non-zero local observables $G_{q,c}$ form the orthonormal basis of MILA. Note that observables $G_{q,c}$ are local operators of order q: they have been represented as expansions

$$G_{q,c} = \sum_{s_0,s_1,\ldots s_q} g_{q,c}^{s_0 s_1 \ldots s_q} Z_{s_0 s_1 \ldots s_q} \tag{5}$$

in terms of spatially homogeneous finite products of field operators

$$Z_{s_0 s_1 \ldots s_q} = \sum_{j=-\infty}^{\infty} \sigma_j^{s_0} \sigma_{j+1}^{s_1} \cdots \sigma_{j+q}^{s_q}$$

where $s_k \in \{0, +, -, z\}$ and $\sigma_j^0 = 1$. The number of non-zero terms in expansion (5) was found to grow exponentially as $\sim 2.55^q$ (on average). The observables $Z_{s_0 \ldots s_q}$ form a convenient orthonormal Euclidean basis w.r.t. (2) of the Hilbert space $\mathfrak{H}$ of local spatially homogeneous observables. Let us now consider truncated linear subspaces of MILA, $\mathfrak{S}_p = \bigcup_{n=0}^{p} (\mathrm{ad}\mathfrak{s})^n \mathfrak{s}$, with dimensions $d_p = \dim \mathfrak{S}_p$, linearly spanned by observables $G_{q,c}$ up to maximal order p, $q \leqslant p$. Let $\boldsymbol{H}_{p,\alpha}$, $\alpha = 0, 1$, denote real and symmetric (Hermitian in general) matrices of linar maps $\mathrm{ad}H_\alpha$ on $\mathfrak{S}_p$ with images orthogonally projected back to $\mathfrak{S}_p$. It follows from the construction that they have (generally) a block-banded structure where the blocks correspond to observables with fixed order q: namely $(G_{q,c}|\mathrm{ad}H_\alpha|G_{q',c'}) \neq 0$ only if $|q - q'| = 1$.

Our aim is to solve equation (3) numerically in truncated subspaces of MILA, $\mathfrak{S}_p$, and check whether the procedure converges as p increases. This is quite feasible since the dimensions $d_p = \dim \mathfrak{S}_p$ increase much less rapidly than, say, the dimensions of truncated subspaces of a huge Lie algebra $\mathfrak{H}$ of homogeneous observables, spanned by $Z_{s_0 \ldots s_p}$. In the case of the KtV model the former increase approximately as $d_p \approx 1.68^{p-1}$ (see table 2) while the latter goes as $\sim 4^{p+1}$. The truncated adjoint maps, $\boldsymbol{H}_{p,\alpha}$, have non-trivial null spaces $\mathfrak{N}_{p,\alpha} = \{A \in \mathfrak{S}_p, [H_\alpha, A] \in \mathfrak{S}_{p+1} - \mathfrak{S}_p\}$, with dimensions $d_{p,\alpha} = \dim \mathfrak{N}_{p,\alpha}$ which increase approximately with the same exponent $\propto 1.68^p$. By means of extensive computer algebra we managed to go as high as $p = 14$. An important observation was that a matrix

$$1 - \exp(\mathrm{i}\tfrac{1}{2}t\boldsymbol{H}_{p,1}) \exp(\mathrm{i}V\boldsymbol{H}_{p,0}) \exp(\mathrm{i}\tfrac{1}{2}t\boldsymbol{H}_{p,1})$$

possesses a high-dimensional null space $\mathfrak{N}_p(t, V)$ whose dimension is, for odd p, *independent* of parameters t, V and equal to the dimension of nullspace of $\boldsymbol{H}_{p,1}$, $\dim \mathfrak{N}_{2l-1} = d_{2l-1,1}$. Note also that for an odd order of truncation $p = 2l - 1$, the elements of null space $A \in \mathfrak{N}_p(t, V)$ are spanned by combinations of *odd* powers of generators, i.e. $(A|G_{2l,c}) \equiv 0$, which are due to the time-symmetric construction of an evolution operator $\hat{U}_{\mathrm{ad}}$. However, we are seeking for LQI $A \in \mathfrak{S}$, which are normalizable, i.e. the *relative norm* in the subspace of local operators of fixed order p, defined as $N_q(A) := \sum_c |(A|G_{q,c})|^2$, should be a rapidly decreasing function of the order q, since $(A|A) = \sum_{q=1}^{\infty} N_q(A) < \infty$. Only the elements of $\mathfrak{N}_p(t, V)$ which are within certain numerical accuracy independent of the order of truncation p are candidates for LQI. To find them we minimize a quadratic form, the relative norm at truncation order $N_p(A)$, i.e. we diagonalize the operator $\hat{N}_p = \sum_c G_{p,c} \otimes G_{p,c}$ in the subspace $\mathfrak{N}_p(t, V)$. In figure 1 we plot the relative norm $N_q(A_m)$ of the first three eigenvectors A_m of $\hat{N}_p$ with $p = 13$ corresponding to the smallest eigenvalues of the quadratic form $N_p(A_m)$ against the (odd) order $q \in \{1, 3, \ldots, p\}$. We give a mesh of plots for various values of the parameters t, V. Note that $N_{2l}(A_m) \equiv 0$ due to symmetry. In a certain region of parameter space t, V we have found a good agreement with *exponential*

$$N_{2l-1}(A_m) \propto \exp(-s_m l) \tag{6}$$

with a positive exponent $s_m(t, V)$ for the first observable $m = 1$, and in a smaller subregion of parameter space even for the second observable $m = 2$, with a smaller exponent s_2. Positivity of s_m as determined from exponential fit (6) for $2l - 1 = 3, 5 \ldots p - 2$ has been used as a numerical criterion of convergence (locality) of A_m. By making a comparison with results for smaller truncation order, $p = 11$, we have checked that the converged observables are (almost) independent on the variation of the truncation order p. For the rest of the null space $\mathfrak{N}_p(t, V)$, $m > 2$, we found a roughly uniform distribution $N_{2l-1}(A_m) \sim 1$ so these observables cannot converge as $p \to \infty$. We conclude that this is evidence for the existence of one or two LQI in certain region in the parameter space, where $t < t_c(V) \approx 1.5$. Outside this region, all exponents $s_m(t, V)$ are zero and none of the eigenvectors A_m converges to a LQI which is consistent with the property of quantum ergodicity. In figure 2 we show a phase diagram of an exponent $s_1(t, V)$ of LQI $A_1(t, V)$. In table 1 we give explicitly the first few coefficients $f_{p,c}$, of an expansion $A = \sum_{(p,c)} f_{p,c} G_{p,c}$ of the one LQI for $t = V = 1$, and of the two LQI for $t = 0.2$, $V = 1$†.

Further, we use a theorem of Mazur [5] generalized by Suzuki [6] (MS) to compute canonical averages‡ of time-averaged correlation functions of certain observables. We

† Of course, at present we cannot completely exclude the possibility that such expansions are generically divergent asymptotic series.

‡ Our example corresponds to infinite temperature since we use a simple trace measure (2).

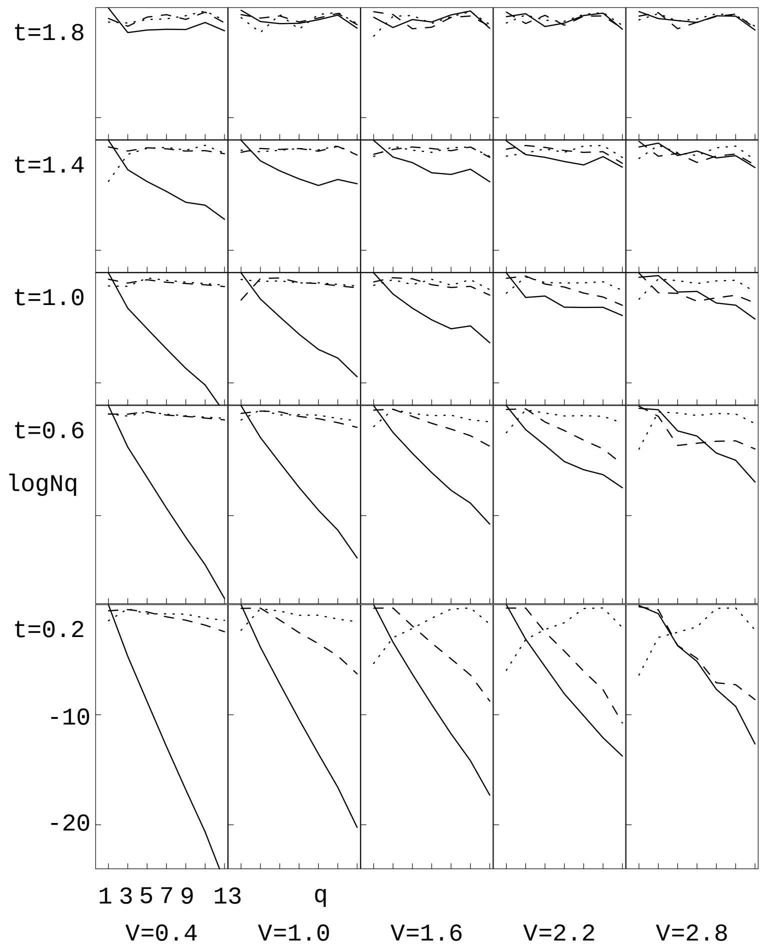

Figure 1. The logarithmic relative norm of the first three candidates $A_m, m = 1, 2, 3$ for LQI, $\log_{10} N_q(A_m)$, is plotted against (odd) order $q = 2l - 1$, for a square mesh of parameters t and V. The order of truncation is $p = 13$. Full curves: $m = 1$, broken curves: $m = 2$, and dotted curves: $m = 3$.

consider dimensionless kinetic energy $T = H_1$, where $\langle T \rangle = (1|T) = 1$, with averaged time correlator

$$D = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} ([T(0) - \langle T \rangle]|[T(n) - \langle T \rangle]) = (T|\bar{T}) - \langle T \rangle^2$$

where $\bar{T} = \lim_{N\to\infty}(1/N)\sum_{n=0}^{N} T(n)$ is a time average of observable T. The MS equation expresses D in terms of a sum over all LQI

$$D = \sum_m |(A_m|T)|^2/(A_m|A_m). \tag{7}$$

In the case of complete integrability there are *infinitely* many LQI, in the case of quantum ergodicity there are *none* and the MS correlator D is zero, while in intermediate regime the sum (7) (for KtV) has one or two non-zero terms. The reader should observe that, according to equation (7), the existence or non-existence of LQI having non-zero overlap with certain

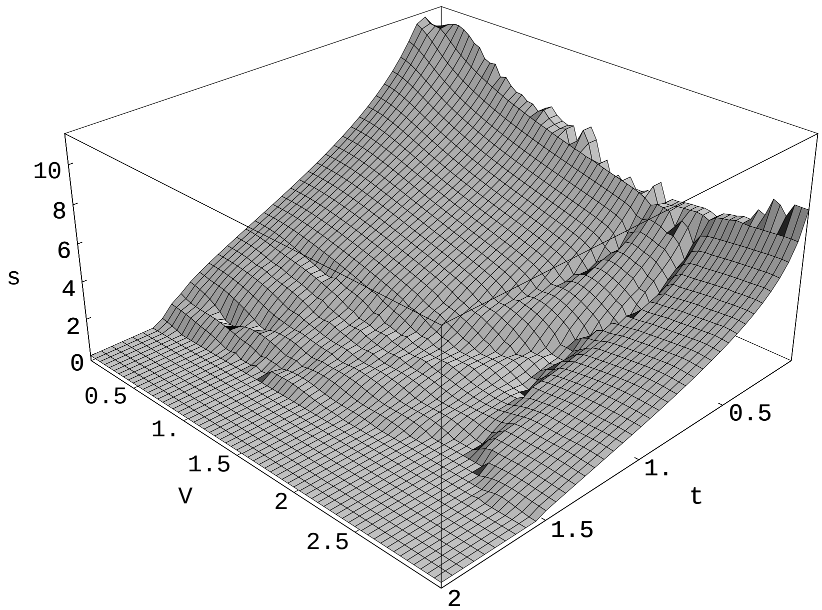

Figure 2. The largest inverse order-localization length s_1 for the first numerical LQI A_1 ($p = 13$) versus parameters t, V.

Table 1. Coefficients of expansion of the two LQI (columns 2, 3) for $t = 0.2$, $V = 1$ and of the single LQI (column 4) for $t = V = 1$ in terms of basis $G_{q,c}$ (column 1, c is a binary code) up to sixth order. Truncation order is $p = 13$. Note that all the digits shown (except possibly the last one) are the same for truncation at $p = 11$. Other obsevables $G_{q,c}$, $q = 1, 3, 5$, are zero by construction (4).

Term	$A_1(t = 0.2)$	$A_2(t = 0.2)$	$A_1(t = 1)$
$G_{1,0}$	−0.965 269 3	−0.186 26	−0.611 62
$G_{1,1}$	−0.260 984 5	0.657 67	−0.788 43
$G_{3,001}$	0.011 624 6	−0.699 08	0.041 95
$G_{3,101}$	0.002 649 9	−0.027 56	0.049 28
$G_{5,01001}$	2.487×10^{-4}	0.200 08	0.005 07
$G_{5,01101}$	4.200×10^{-5}	0.003 37	0.004 50
$G_{5,11001}$	4.533×10^{-5}	0.008 65	0.004 87
$G_{5,11101}$	9.884×10^{-6}	0.000 56	0.005 55

Table 2. Dimensions of truncated MILA $\mathfrak{S}_p$, and of nullspaces of $\boldsymbol{H}_{p,\alpha}$, for different orders of truncation p.

p	2	3	4	5	6	7	8	9	10	11	12	13	14
d_p	3	5	7	11	16	26	41	67	108	179	294	495	832
$d_{p,0}$	1	2	3	5	6	10	13	23	34	61	92	163	258
$d_{p,1}$	1	2	1	5	2	10	7	21	22	51	66	137	202

dynamical observable T corresponds to the non-ergodicity or ergodicity of T, respectively. However, analysis based solely on LQI cannot give us any further information on time-

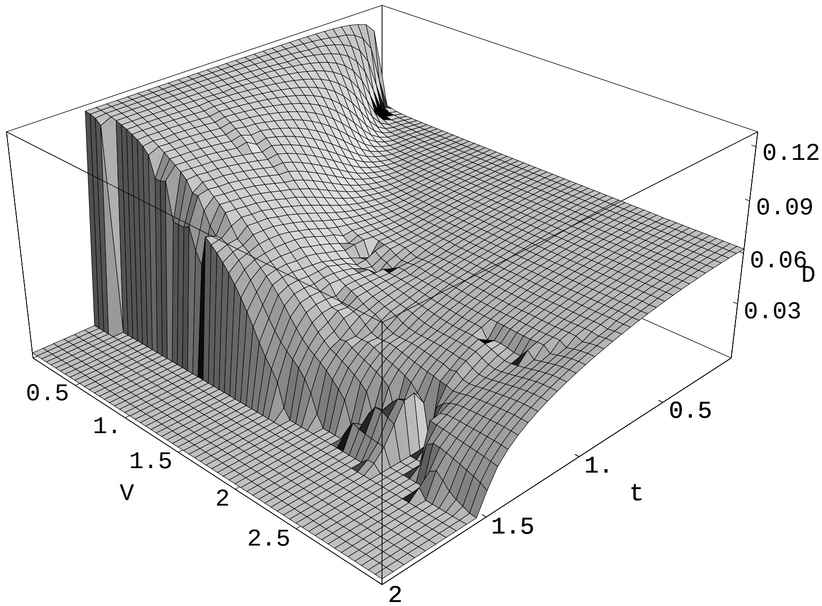

Figure 3. MS correlator (7) D ($p = 13$) versus parameters t, V.

correlation functions and the stronger property of quantum mixing [1, 4, 7]. In figure 3 we show a phase diagram of the kinetic time-correlator D as determined from LQI (7). Note a sharp (phase) transition between ergodic dynamics (disordered phase $D = 0$) and intermediate dynamics (ordered phase $D > 0$) which may also be characterized by the maximal inverse *order-localization length* s_1 (figure 2) which linearly decreases to zero when approaching the critical curve $t_c(V)$.

We have compared the above results on infinite KtV chains with direct calculations on finite chains of L sites with periodic boundary conditions, $c_{L+1} = c_1$, which have a discrete quasi-energy spectrum η_n and eigenstates $|n\rangle$, $U|n\rangle = \exp(-\mathrm{i}\eta_n)|n\rangle$. For a finite system of size L, a time-correlator reads

$$D_L = 2^{-L} \sum_{n=1}^{2^L} \left(\frac{1}{L} \langle n|T|n\rangle - \langle T\rangle \right)^2 . \qquad (8)$$

However, D_L need not necessarily converge to the proper time correlator of an infinite system $L = \infty$, since the time-limit $\tau \to \infty$ (implicit in (8)) is taken prior to the size-limit $L \to \infty$, whereas in general the two limits do not commute [1]. Nevertheless the behaviour of D_L for $L = 16$ shown in figure 4 as a function of parameters t, V is very similar to the MS correlator D of an infinite system (7) shown in figure 3. Agreement is even quantitative, except in the region of the transition between dynamical phases (figure 5).

Note that the KtV map $\hat{U}_{\mathrm{ad}}$ is invariant under the parity operation $\hat{P} : c_j \to c_{-j}$ and MILA $\mathfrak{S}$ may exhaust† only the positive parity class of observables, $\hat{P}A = A$. Unfortunately, the *particle current* $J = \mathrm{i}(\sum_j c_j^\dagger c_{j+1} - \mathrm{h.c.})$, which, interestingly, gives rise to ideal (ballistic) transport in intermediate [4] (integrable [2, 3]) regime, has a

† We have also tried to search for LQI in the entire (huge) algebra of homogeneus observables $\mathfrak{H}$ (truncated at order $p = 6$), i.e. we have numerically solved equation (3) for the matrix of the map U_{ad} in the basis of observables $Z_{s_0 \ldots s_p}$, and we found roughly identical (and no more) numerical LQI as in MILA, although $\mathfrak{S} \subset \mathfrak{H}$.

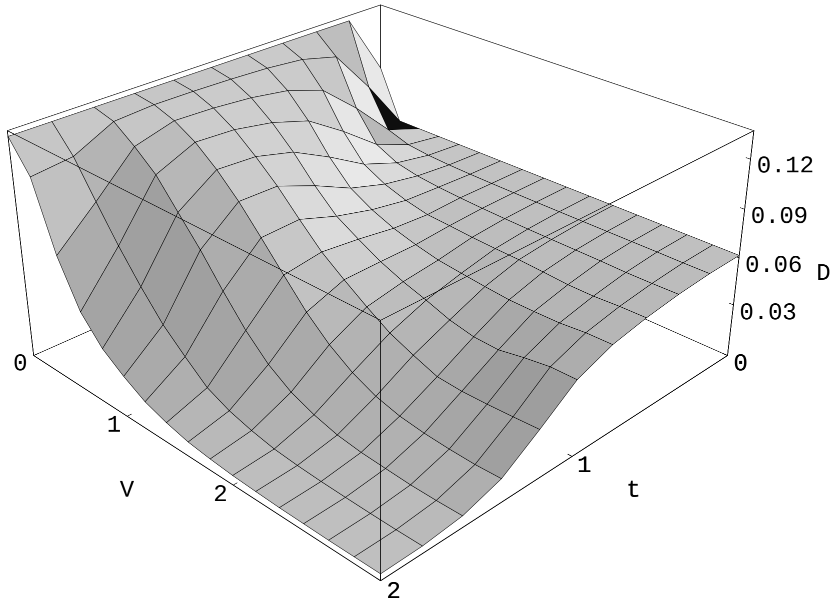

Figure 4. The finite-size kinetic correlator (8) D_L for $L = 16$ versus parameters t, V.

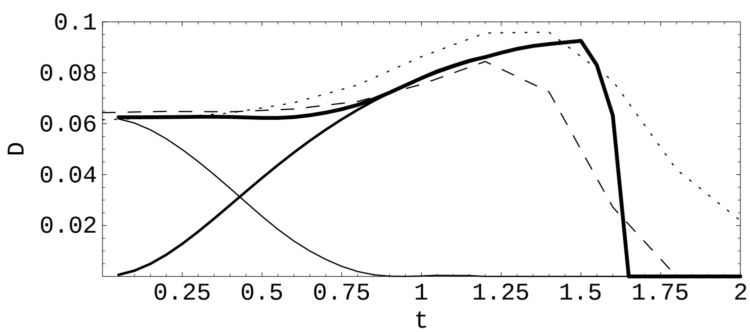

Figure 5. The comparison between MS correlator (7) D of infinite KtV chain (heavy full curve) and finite-size correlator (8) D_L for $L = 16$ (dotted curve) and linearly extrapolated to $1/L = 0$ from data for $L = 16$ and $L = 12$ (broken curve) versus parameter t, and for fixed parameter $V = 1$. Medium full curve and light full curve denote separate contributions of the first and second LQI to MS correlator D (7), respectively.

negative parity $\hat{P}J = -J$, and hence zero overlap with the above LQI of MILA, $(A_m|J) = (\hat{P}A_m|\hat{P}J) = -(A_m|J) = 0$. However, similar results have been found recently for the negative parity class of observables as well [7].

The total occupation number $N = \sum_j n_j = \frac{1}{2}\sum_j(\sigma_j^z + 1)$, is a trivial invariant of motion since it has zero overlap with dynamical observables, $(N|\mathfrak{S}) = (N|J) = 0$. One may either consider observables over the Fock subspace of states with a fixed density $\rho := \langle N \rangle \in [0, 1]$ (which is an additional external parameter), or as we do here using a full trace average (2), consider the entire Fock space, which is in TL equivalent to a half-filled lattice, $\rho = \frac{1}{2}$.

We have found strong evidence for the existence of non-trivial LQI, in a simple but generic non-integrable quantum many-body system in TL, namely a KtV model [4].

The algebraic method, which should be applicable to other non-integrable quantum many-body systems, is based on the (computerized) construction of minimal invariant infinitely dimensional Lie algebra, MILA, generated by the essential parts of the Hamiltonian (in our case, by kinetic energy and kick potential). LQI are found numerically as fixed points of the adjoint map of the evolution operator (or of the Hamiltonian if system was autonomous) in MILA. The existence of LQI is found to be fully consistent with deviations from quantum ergodicity characterized by non-vanishing averaged time-autocorrelations D of a typical observable; here we use the kinetic energy. D is a suitable order parameter describing the phase transition from the quasi-integrable (intermediate) regime ($D > 0$) to the quantum ergodic regime ($D = 0$).

Numerical results on quantum transport in a different model, namely a non-integrable extended Hubbard chain, have recently been reported [8] which are in agreement with the above conjecture.

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